Annex II (to parts 3 and 4)

Model analysis of the difference between two sorts of mean temperatures, $(T_m)_A$ = GMT and $(T_e)_g = T_s * (A_{proj}/A)^{0,25}$ = bolometric mean, additionally referred to earth's moon and earth herself

In order to try and arrive at an appropriately understandable description of physically unusual facts we are not content to use words alone but rather to include model analyses, too, wherever possible. So it is with the difference between 2 global T-means, which are obtained by different methods. The following example, **Figure II.1**, refers to a model case close enough to reality, consisting of a "celestial stone ball" *without any horizontal heat transfer on its surface*, which circles the sun in synchronous rotation thus showing her permanently the same part of its surface. Rotation occurs around an axis being vertically oriented to the plane of the orbit. What we now want to determine, is the relation $\tau = (T_m)_A / (T_e)_g$.

We take from the figure: $\Delta A_i = \Delta l * 2\pi * y_i$ and $(\Delta A_i)_{proj} = \Delta y_i * 2\pi * y_i$.

If we now consider $\Delta y_i = \Delta l * \sin \alpha_i$, we obtain the relation

$v = \Delta y_i / \Delta l = \sin \alpha_i = (\Delta A_i)_{proj} / \Delta A_i$

which is to be introduced in the much-cited equation

$$(T_{e,i})^4 = S*(1- a)/\sigma * A_{proj} / A \qquad \text{(IX.3, see part 3)}$$

together with $T_s = [S*(1- a)/\sigma]^{0,25}$ at the sub-solar point, P_s. We thus obtain the abbreviated form,

$$T_{e,i} = T_s * (\sin \alpha_i)^{0,25}.$$

Imagine there were a documented number, n, of $(T)_{A,i}$,(i = 1, 2,…, n) resulting from in situ T-measurements on the sunlit hemisphere which are allocated to an equivalent number of (circular) zones completely covering the hemisphere. Imagine furthermore, a researcher was ordered to determine – on the basis of the numerous $(T)_{A,i}$ – the area-weighted T-average, i.e. the $(T_m)_A$ for the bright hemisphere, by following a rule in the form presented by equation (V), putting there $A=2\pi*R^2$, i.e. the half of a sphere's surface.

In case the researcher was detecting now a good correlation between the $(T)_{A,i}$ measured and the $(T_e)_i$ calculated, – thus allowing for $(T)_{A,i} \cong (T_e)_i$ – he almost certainly would think to fulfil his task in a merely analytical, unassailable way, as follows:

First, the subdivision into circular zones around P_s has to be made infinitely fine (we of course do not mind our imagining an infinite number of measuring stations which had to comply with that number theoretically!). Hence we arrive at the integral equation (indices now becoming obsolete):

$$A*(T_m)_A = \int T_e \, dA \, .$$

This connection has to be seen entirely in analogy to equation (V), which was mentioned as a means for determining a MGT on the earth's ground.

As indicated above we must now write $T_e = T_s * (\sin \alpha)^{0,25}$. In order to solve the integral, $dA = dl*2\pi*y$ also has to be formulated as a function of α. Because there is

$y = R* \cos(\pi/2 - \varphi) = R* \cos \alpha$, and also $d\ell = - R*d\alpha$

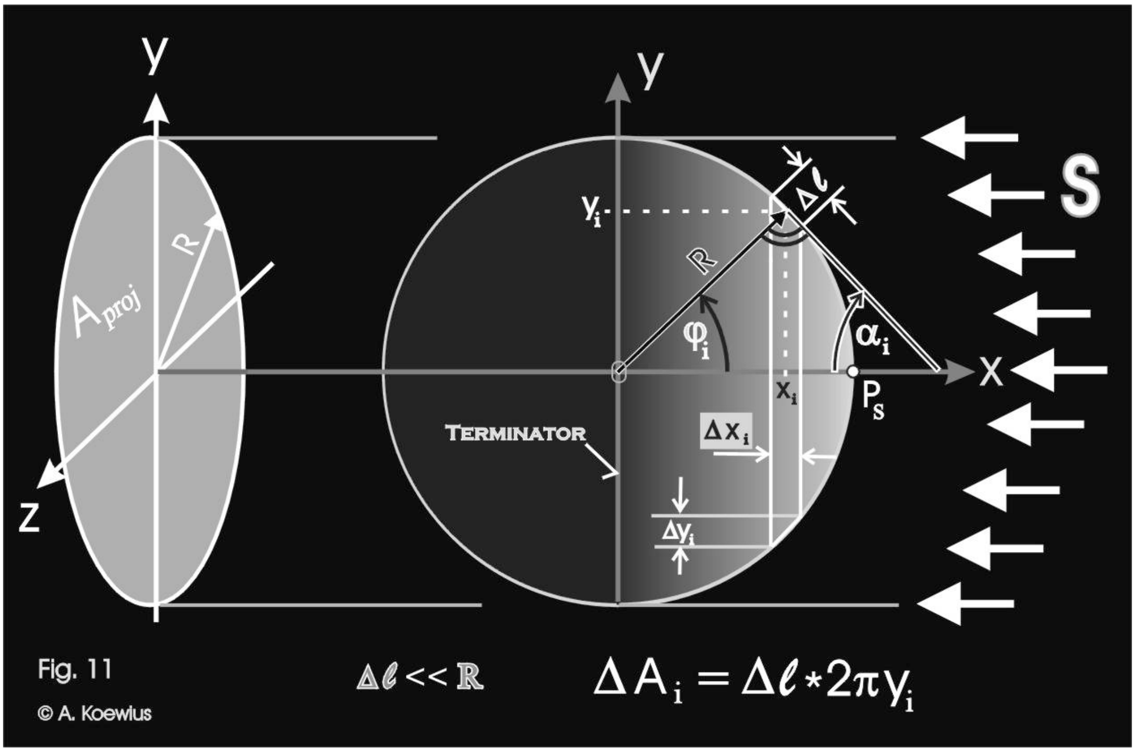

Figure II.1: Illustration of how to determine an area-weighted temperature mean value $\mathbf{(T_m)_A}$ by referring to a celestial model case sufficiently close to reality. The model consists of a planetary stone ball without an atmosphere, which circles the sun in synchronous rotation, its rotational axis being assumed as vertically oriented to the plane of the orbit. If one looks at one of the circular zones (on the sunlit hemisphere), showing the small width $\Delta\ell_i$, it becomes obvious that the angle of inclination, α_i , must be the same at any spot within that zone. The surface area of the i^th^ zone is $\Delta A_i = \Delta\ell * 2\pi * y_i$. Since we have no significant horizontal heat transfer on the globular surface, temperatures on the sunlit ground lie in between T_s (at the sub-solar point, P_s) and T = 0 °K on the terminator. The latter designation means that great circle which separates the bright hemisphere from the dark one. T = 0 °K is, of course, also met on the dark side of the sphere. There are as many T-measurements to be taken "in situ" as there are circular zone areas. It is presupposed for every circular area i, that the measured T-value corresponds to $\mathbf{T_{e,i} = T_s * (\sin \alpha_i)^{0.25}}$.

[resulting from $\mathbf{dy = - R * \sin \alpha * d\alpha = d\ell * \sin \alpha}$],
we have to put $\mathbf{dA = -2\pi * R^2 * \sin \alpha\, d\alpha}$, thus obtaining

$$\int T_e\, dA = -2\pi * R^2 * T_s \int (\sin \alpha)^{0,25} * \cos \alpha\, d\alpha$$

with $\alpha = 0$ as the upper limit and $\alpha = \pi/2$ as the lower limit of that *definite* integral. By means of the substitution $\sin \alpha = u$ we quickly arrive at

$$\int T_e\, dA = 8/5 * \pi\, R^2 * T_s\,.$$

Because of $\mathbf{A = 2\pi * R^2}$ we eventually get

$$(T_m)_A = 4/5 * T_s\,.$$

This result has now to be compared with the $(\mathbf{T_e})_g$ after equation (IX.3): Substituting $\mathbf{A_{proj}/A = \pi\, R^2 / (2\pi\, R^2) = ½}$ gives us $(\mathbf{T_e})_g = \mathbf{T_s / 2^{0,25}}$. So both values calculated lead to the quotient

$$\tau = (T_m)_A \,/\, (T_e)_g = 0{,}8 * 2^{1/4} = 0{,}9514.$$

In other words, $(T_m)_A$ in our model case is about 5 % lower than $(T_e)_g$ calculated on the basis of an overall radiation budget.

But let us first take the **moon** as one of those examples, (which our model case should apply to as a good approximation) by considering the following: **(a)** The lunar rotational velocity relative to the sun is small. It amounts to only ~ 360°/30 days = 12° per one day on earth. So the sun walks about 30 times more slowly across the "lunar sky" for a certain point on the globular surface than she does on earth. **(b)** Moon's rotational axis is nearly (but not quite exactly) perpendicular to the S-direction. Both circumstances lead to the facts as shown in **Figure II.2**: So the temperatures on the dark hemisphere lie around 100 to 125 K (instead of 0 K like in our model case) thus leading to an infrared radiative flux density of only around 14 W/m^2 which emerges from the dark lunar surface. Hence our T_s-value of 387 K, calculated for the bright hemisphere alone, is left practically unaffected.

We had already calculated $(T_e)_g$ = 325 K = 52 °C for the sunlit hemisphere of the moon. Hence an area-weighted mean, $(T_m)_A$, would attain some 325∗0.9514 = 309 K = 36 °C, which is not all together satisfactory since only $(T_e)_g$ has a useful physical meaning.

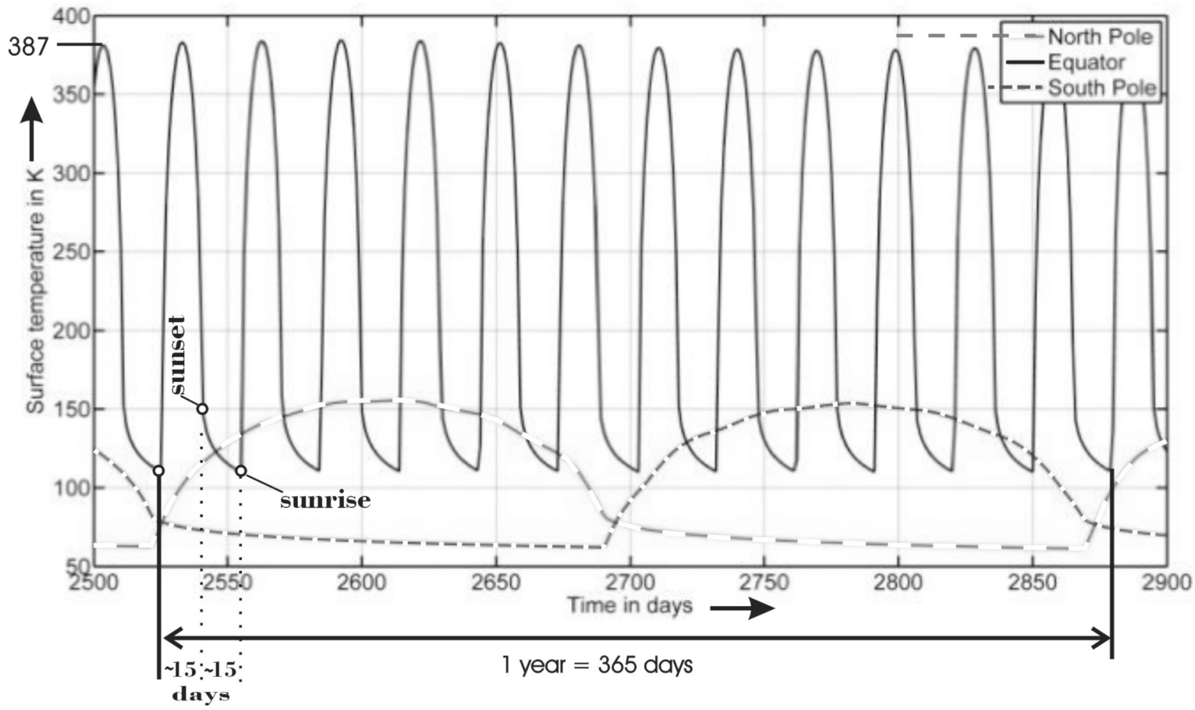

Figure II.2: Temperature cycles on moon's surface at 3 different spots; one on the equator (duration of one cycle: one month) and one at each pole (duration of one cycle: ≈ one year, due to a slight inclination of moon's rotational axis against the ecliptic). T = 387 K means T_s. (after K. Bauch et al., Muenster Univ.: „Estimation of lunar temperatures: a numerical model", complemented by Koewius)

In the light of these considerations we should at last dedicate some remarks to the **earth**. First, on the terrestrial surface a horizontal heat exchange (resulting from flows of air and ocean waters) takes place to such an extent that regionally established annual **T**-means differ maximally between ~27 °C (in the equatorial region) and

~ – 25 °C (in the polar regions), i.e. we get the difference of **ΔT** ~ 52 degrees, as an order of magnitude. On the moon we meet **ΔT** ~ 270 degrees as the difference which exists between $\mathbf{T_s}$ (at the sub-solar point) and **T** on the terminator (and on the dark side of the moon, respectively). Secondly the earth rotates so fast that – within 24 hours – there is only a difference of some 10 degrees between day and night or an amplitude of 5 degrees around MGT = 15 °C according to the "earth fact sheet" of NASA.

So, if we find only a relatively small difference between $\mathbf{(T_e)_g}$ and $\mathbf{(T_m)_A}$ as in the case of the moon, then we are inclined more to set $\mathbf{(T_e)_g}$ equal to $\mathbf{(T_m)_A}$ in the case of the earth. We have, however, yet to clear up the question of a value for $\mathbf{(T_e)_g}$, which can correspond to our $\mathbf{(T_m)_A}$ on the ground. For this purpose we set **a** = 0.3 in

$$\mathbf{(T_e)^4_g = S * (1- a)/\sigma * A_{proj} / A} \quad \text{(IX.3)},$$

which is the official value for the albedo of today's earth. Secondly we introduce there **–** as the radiating surface **–** $\mathbf{A = 4\,\pi * R^2}$ (being the <u>whole</u> surface of a sphere in accordance with the above discussion). Hence we obtain – together with S_E = 1367 W/m^2 – the much cited value $\mathbf{(T_e)_g}$ = 255 K = - 18 °C. So it is this **T**-value which must be set equal to $\mathbf{(T_m)_A}$ = MGT which would result from measurements on the ground in that fictitious case of reference calling for the total absence of greenhouse gases of any kind in the atmosphere. As is well known, it is the natural greenhouse effect – based upon CO_2 (**k*** = 280 ppm) and water vapour in the air – which transforms a chilly GMT = - 18 °C into life-friendly MGT* = + 15 °C, hence rising the global **T**-mean by **ΔT*** = + 33 degrees.

Consequently the question arises "Can the difference between the temperatures on the equator and at the poles, we quoted above, be also transferred to the fictitious case 'MGT = $\mathbf{(T_e)_g}$ = - 18 °C' by simply subtracting **ΔT** = 33 degrees everywhere on the ground? And is there any evidence visible to do so?" We mean 'yes'. Since it were possible, in the case of CETGs being totally absent, to confirm $\mathbf{(T_e)_g}$ by measurements directly on the ground, the attempt seems profitable to calculate (according to equation (IX.3)) at least the temperature $\mathbf{T_e}$ for a narrow equatorial zone provided that the sun stands – more or less – directly over the equator. We call $\Delta\ell$ the width and $\mathbf{s = 2\pi * R}$ the length of that zone (with R as the radius of the earth = 6371 km). Hence we obtain $\mathbf{A = \Delta\ell * 2\pi * R}$ and $\mathbf{A_{proj} = \Delta\ell * 2 * R}$. Together with **a** = 0.3 and $\mathbf{S_E}$ = 1367 W/m² we arrive at $\mathbf{(T_e)_{equator}}$ **=** 271 K **=** -2 °C. And this value eventually leads to the difference + 27 °C – (- 2 °C) = 29 degrees (instead of 33). So the difference already lies in the right order of magnitude. However we have to consider that the value of
-2 °C (= 271 K) was derived from a calculation which is exactly valid only for the total absence of any heat transfer parallel to the planetary surface (in our terrestrial case: in the direction towards higher latitudes). This situation may apply to "stone balls" like the moon, however by no means the earth and her atmosphere *regardless* of a mean temperature being 29 or 33 degrees lower than a MGT which is met on earth in reality. So, in this light, the said -2 °C represents nothing more than an upper limit with regard to the equatorial zone (in the fictitious case).